Multiple Comparisons (Sleuth3 Sections 6.3 and 6.4)

Example 1: Diet restriction and longevity in mice (Sleuth3 Case study 5.1.1)

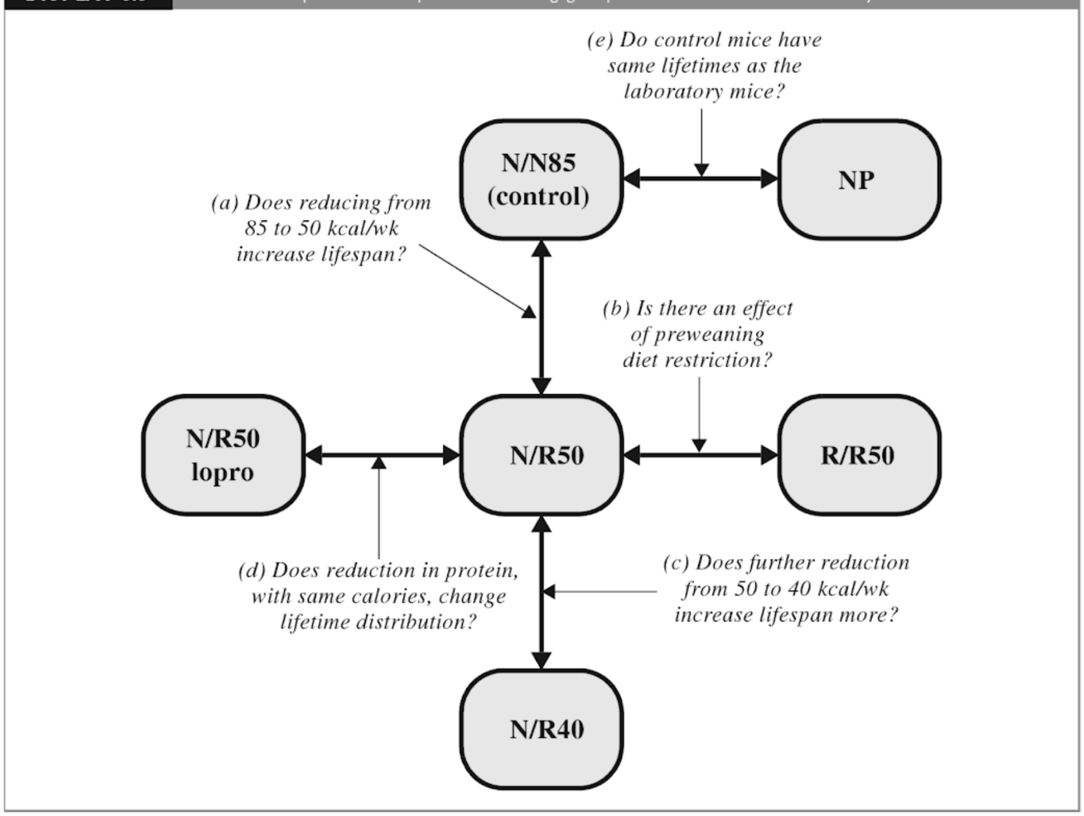
Mice were randomly assigned to one of 6 treatment groups with different diets to investigate relationships between diet and lifetime. The life span of each mouse was recorded in months.

1. **NP**: Mice ate as much as they wanted of standard food for lab mice
2. **N/N85**: Control group. **N**: no intervention before weaning; ate as normal. **N85**: no intervention after weaning; fed weekly diet of 85kcal/week (standard diet for lab mice)
3. **N/R50**: **N**: no intervention before weaning. **R50**: after weaning, restricted diet of 50 kcal/week
4. **R/R50**: **R**: restricted diet of 50 kcal/week before weaning. **R50**: after weaning, restricted diet of 50 kcal/week
5. **N/R50 lopro**: **N**: no intervention before weaning. **R50**: after weaning, restricted diet of 50 kcal/week. Dietary protein decreased with mouse age.
6. **N/R40**: **N**: no intervention before weaning. **R40**: after weaning, restricted diet of 40 kcal/week

Denote the mean life spans in the population of mice fed each of these diets under laboratory conditions by μ_1 through μ_6.

Planned Comparisons: Before data were collected, researchers decided on the comparisons below:

DISPLAY 5.3 Structure of planned comparisons among groups in the diet restriction study



(a) Are the population mean lifetimes the same for the **N/N85** and **N/R50** groups?
   - Confidence interval for $\mu_2 - \mu_3$ or test of $H_0 : \mu_2 = \mu_3$ vs $H_A : \mu_2 \neq \mu_3$.

(b) Are the population mean lifetimes the same for the **N/R50** and **R/R50** groups?
   - Confidence interval for $\mu_3 - \mu_4$ or test of $H_0 : \mu_3 = \mu_4$ vs $H_A : \mu_3 \neq \mu_4$.

(c) Are the population mean lifetimes the same for the **N/R50** and **N/R40** groups?
   - Confidence interval for $\mu_3 - \mu_6$ or test of $H_0 : \mu_3 = \mu_6$ vs $H_A : \mu_3 \neq \mu_6$.

(d) Are the population mean lifetimes the same for the **N/R50** and **N/R50** lopro groups?

- Confidence interval for $\mu_3 - \mu_5$ or test of $H_0 : \mu_3 = \mu_5$ vs $H_A : \mu_3 \neq \mu_5$.

(e) Are the population mean lifetimes the same for the **N/N85** and **NP** groups?

- Confidence interval for $\mu_2 - \mu_1$ or test of $H_0 : \mu_2 = \mu_1$ vs $H_A : \mu_2 \neq \mu_1$

Example 2: Handicaps and hiring (Sleuth3 Case Study 6.1.1 in Sleuth 3)

A 1990 study conducted a randomized experiment to explore how physical handicaps affect people's perception of employment qualifications. The researchers prepared five videotaped job interviews using the same two male actors for each. A set script was designed to reflect an interview with an applicant of average qualifications. The videos differed only in that the applicant appeared with a different handicap:

1. in one, he appeared to have no handicap;
2. in a second, he appeared to have one leg amputated;
3. in a third, he appeared on crutches;
4. in a fourth, he appeared to have impaired hearing;
5. and in a fifth, he appeared in a wheelchair.

Seventy undergraduate students from a US university were randomly assigned to view the videos, fourteen to each video. After viewing ther video, each subject rated the qualifications of the applicant on a 0 to 10 point applicant qualification scale.

Denote by μ_1 through μ_5 the mean qualification score in the population of ratings that might be given by US undergraduate students from the US university in this study for each of the 5 handicaps groups.

"Unplanned" Comparisons: Maybe we want to compare the mean qualification score for every pair of groups

- Confidence interval for $\mu_1 - \mu_2$ or test of $H_0 : \mu_1 = \mu_2$ vs $H_A : \mu_1 \neq \mu_2$
- Confidence interval for $\mu_1 - \mu_3$ or test of $H_0 : \mu_1 = \mu_3$ vs $H_A : \mu_1 \neq \mu_3$
- Confidence interval for $\mu_1 - \mu_4$ or test of $H_0 : \mu_1 = \mu_4$ vs $H_A : \mu_1 \neq \mu_4$
- Confidence interval for $\mu_1 - \mu_5$ or test of $H_0 : \mu_1 = \mu_5$ vs $H_A : \mu_1 \neq \mu_5$
- Confidence interval for $\mu_2 - \mu_3$ or test of $H_0 : \mu_2 = \mu_3$ vs $H_A : \mu_2 \neq \mu_3$
- Confidence interval for $\mu_2 - \mu_4$ or test of $H_0 : \mu_2 = \mu_4$ vs $H_A : \mu_2 \neq \mu_4$
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- Confidence interval for $\mu_3 - \mu_4$ or test of $H_0 : \mu_3 = \mu_4$ vs $H_A : \mu_3 \neq \mu_4$
- Confidence interval for $\mu_3 - \mu_5$ or test of $H_0 : \mu_3 = \mu_5$ vs $H_A : \mu_3 \neq \mu_5$
- Confidence interval for $\mu_4 - \mu_5$ or test of $H_0 : \mu_4 = \mu_5$ vs $H_A : \mu_4 \neq \mu_5$

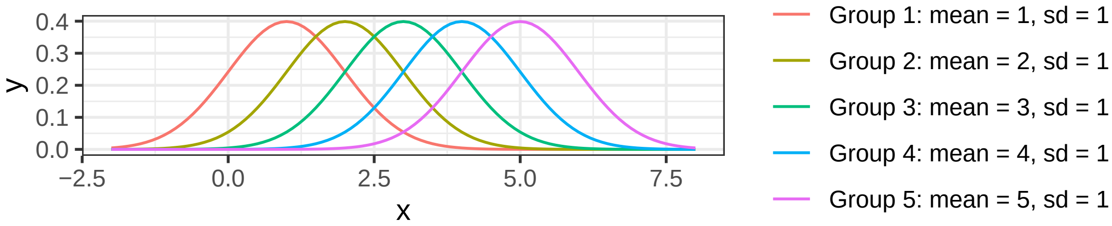
There are 10 different comparisons to do.

Individual Confidence Level vs. Familywise Confidence Level

- Individual confidence level: the proportion of samples for which a single confidence interval contains the parameter it is estimating
- Familywise confidence level: the proportion of samples for which every one of several different confidence intervals contain the parameters they are estimating

Example (simulation study)

Suppose I have 5 groups with means $\mu_1 = 1$, $\mu_2 = 2$, $\mu_3 = 3$, $\mu_4 = 4$, $\mu_5 = 5$ and standard deviation $\sigma = 1$.



Results for 1 simulation

- Simulated a data set with 100 observations from each of the 5 groups
- Calculated 95% confidence intervals for differences in group means, for each pair of means (10 intervals total)

Groups	Difference in Means	95% CI lower bound	95% CI upper bound	Contains true difference?
2, 1	2 - 1 = 1	0.99	1.54	Yes
3, 1	3 - 1 = 2	1.60	2.16	Yes
4, 1	4 - 1 = 3	2.87	3.42	Yes
5, 1	5 - 1 = 4	3.55	4.10	Yes
3, 2	3 - 2 = 1	0.34	0.89	No
4, 2	4 - 2 = 2	1.60	2.15	Yes
5, 2	5 - 2 = 3	2.28	2.84	No
4, 3	4 - 3 = 1	0.99	1.54	Yes
5, 3	5 - 3 = 2	1.67	2.22	Yes
5, 4	5 - 4 = 1	0.41	0.96	No

For this particular sample, 7 out of 10 of the confidence intervals contain the difference in means they are estimating.

Repeated for 1000 simulations:

- Repeated the process above for 1000 different simulated data sets. Table shows:
  - percent of samples for which each CI comparing 2 groups succeded
  - percent of samples for which all 10 CIs succeeded

Basic idea: Make individual confidence levels larger to get desired familywise confidence level.

Groups	Percent of Samples Successful
2, 1	95.1%
3, 1	94.5%
4, 1	95.0%
5, 1	94.5%
3, 2	95.5%
4, 2	95.1%
5, 2	94.8%
4, 3	94.9%
5, 3	95.7%
5, 4	94.4%
All 10 comparisons	71.1%

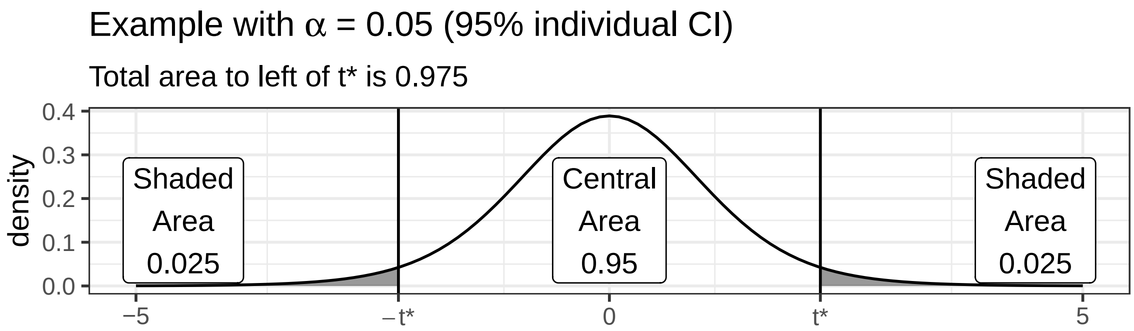
Bonferroni adjustment

- Intuition with 10 intervals:
  - Familywise confidence level 95%: for 95% of samples, all 10 intervals should simultaneously contain the parameter they are estimating.
  - For 5% of samples, at least one of the 10 does not contain the parameter it is estimating
  - Each individual CI misses for 0.5% of samples
  - Each individual CI has confidence level 99.5%

Groups	Target Percent of Samples Successful (Confidence Level)	Target Percent of Samples UNSuccessful (100 - Confidence Level)
2, 1		
3, 1		
4, 1		
5, 1		
3, 2		
4, 2		
5, 2		
4, 3		
5, 3		
5, 4		
All 10 comparisons	95% ($1 - \alpha = 0.95$)	

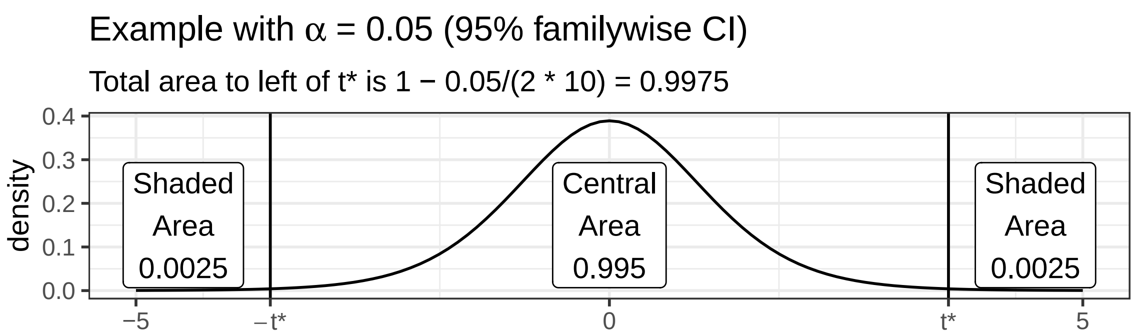
Reminder of procedure for an individual confidence interval

- In this class, all confidence intervals are calculated as Estimate $\pm$ Multiplier $\times$ SE(Estimate)
- So far, the Multiplier is $t_{df}(1 - \alpha/2)$. Example: for a 95% CI, $\alpha = 0.05$, and $1 - \alpha/2 = 0.975$



Individual intervals have higher confidence levels to get desired familywise confidence level

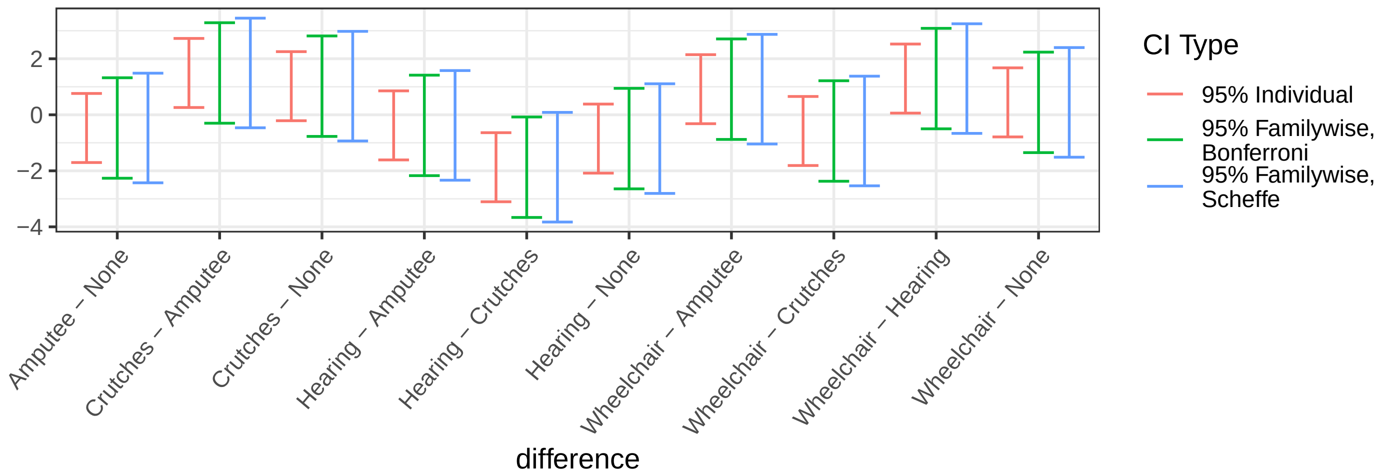
- In general, if there are k confidence intervals to compute, use Multiplier $= t_{df}(1 - \alpha/2k)$



Scheffe adjustment

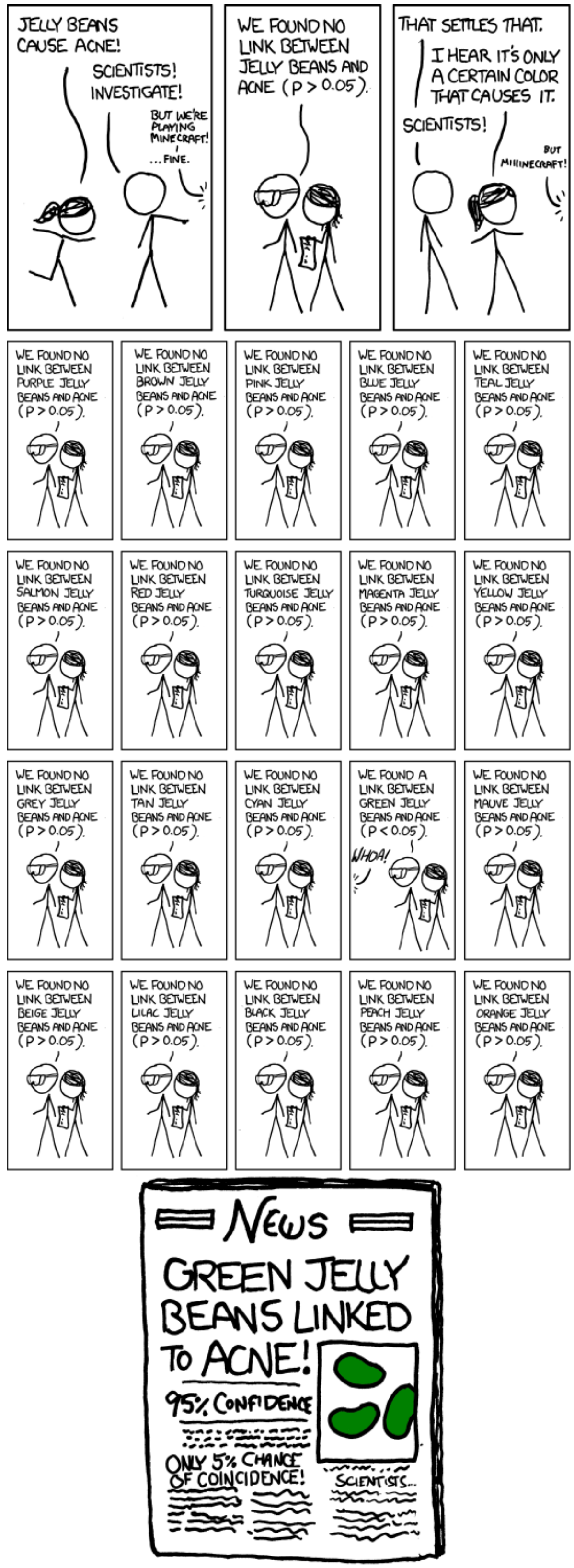
- Use Multiplier = $\sqrt{(I-1)F_{(I-1),(n-1)}(1-\alpha)}$
- Generally a larger multiplier (wider CIs) than the Bonferroni adjustment
- Works for familywise inferences about every possible linear combination of group means $\gamma = C_1\mu_1 + \cdots + C_I\mu_I$
  - (Doesn't matter how many! Same adjustment for any number of intervals in the family!)
- Usually not useful for ANOVA, but very useful for regression models, coming soon!

All 10 CIs plotted for each method



Similar ideas for hypothesis tests

- p-value = probability of obtaining a test statistic at least as extreme as the value of that statistic we got in our sample data, if H_0 is true **in a single test**
- If H_0 is actually correct, 5% of samples will have a p-value < 0.05 **by definition of a p-value**. Imagine we conduct 20 hypothesis tests: (Source: xkcd)



- We need to recalibrate how small a p-value must be to provide evidence against the null hypothesis.

Individual p-value	Strength of evidence against H_0 (one test)	Compare to...	But, Repeated 10 times
0.10 or less	Some evidence; not conclusive	Probability of 4 heads in a row is 0.0625	Probability of 4 heads in a row at least once in 10 repetitions is 0.4755
0.05 or less	Moderate	Probability of 5 heads in a row is 0.03125	Probability of 4 heads in a row at least once in 10 repetitions is 0.2720
0.01 or less	Strong	Probability of 7 heads in a row is 0.007813	Probability of 7 heads in a row at least once in 10 repetitions is 0.0754
0.001 or less	Very strong evidence	Probability of 10 heads in a row is 0.0009766	Probability of 10 heads in a row at least once in 10 repetitions is 0.00972

- The chance of obtaining a small p-value in at least one of the tests is larger than the chance of obtaining a small p-value in a single test.
- Roughly, if I conduct 10 tests a p-value of 0.001 for one of those tests provides the same amount of evidence against the null hypothesis as a p-value of 0.01 if I only did a single test.

A second idea (not perfect)

- Conduct an F test of $H_0 : \mu_1 = \mu_2 = \ldots = \mu_I$ vs H_A : at least one mean is different from the others
  - If this F test gives strong evidence against the claim that all means are equal, proceed to look at individual results, typically using unadjusted intervals/p-values
  - If the F test doesn't give strong evidence against the claim that all means are qual, stop! Even if some individual comparisons had small p-values, you're done.

When to bother?

Opinions differ

- Book says:
  - if tests are "planned", no need to adjust for multiple comparisons
  - if tests are "unplanned", adjust
- Some people say you should always adjust for multiple comparisons
- I say you need to understand the issues and report what you are doing:
  - **Familywise confidence levels can be much less than individual confidence levels**
  - **Report whether or not you have adjusted for multiple comparisons**
  - **Report all confidence intervals/hypothesis tests you perform**, whether or not the results are "statistically significant" (p-value less than some threshold). **Reporting only statistically significant results is cheating.**
  - To the extent possible, **plan your analysis before collecting data**, and keep number of planned comparisons small