Concepts: Transformations for ANOVA models

Sleuth3 Sections 3.5 and 5.5

Context

- Transformations can sometimes help with the following issues:
  - non-normal distributions within each group (but skewness is only a problem if it is very serious)
  - lack of equal variance for all groups
  - outliers (but usually only if this is a side effect of serious skewness)
- The most common transformations (that we'll consider in this class) work for positive numbers only.

The Ladder of Powers

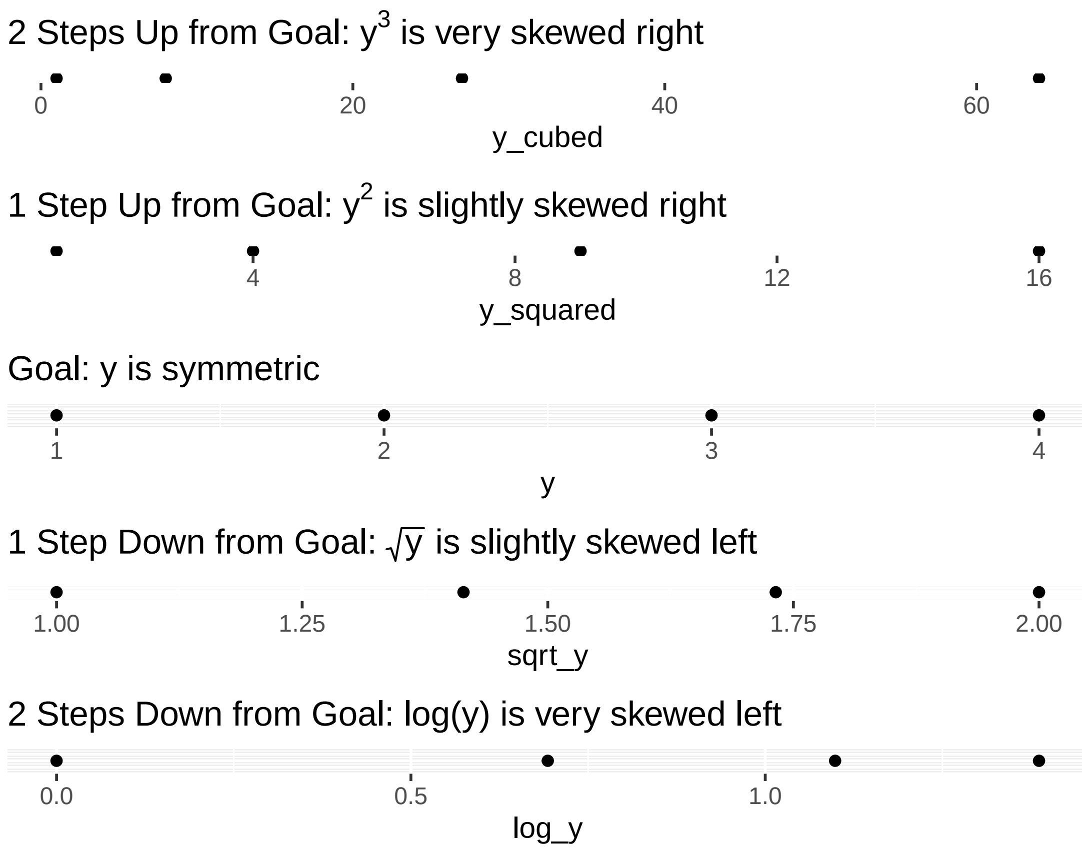
- Imagine a "ladder of powers" of y (or x): We start at y and go up or down the ladder.

Transformation	R Code	Comments
$\vdots$		
e^y	`exp(y)`	Exactly where on the ladder the exponential transformation belongs depends on the magnitude of the data, but somewhere around here...
y^2	`y^2`	
y		Start here (no transformation)
$\sqrt{y}$	`sqrt(y)`	
$y^{\text{“}0\text{”}}$	`log(y)`	We use $\log(y)$ here
$-1/\sqrt{y}$	`-1/sqrt(y)`	The $-$ keeps the values of y in order
$-1/y$	`-1/y`	
$-1/y^2$	`-1/y^2`	
$\vdots$		

Some (minimal) facts about logarithms and exponentials

- Foundations:
  - In this class the base of our logarithms is e
  - Notation: $\exp(x) = e^x$
- log() and exp() are inverses
  - $\log(\exp(x)) = x$
  - $\exp(\log(x)) = x$
- They are useful because they convert multiplication to addition, and addition to multiplication
  - $\log(a \cdot b) = \log(a) + \log(b)$
  - $\exp(a + b) = \exp(a) \cdot \exp(b)$

- Which direction?
  - If a variable is skewed right, move it down the ladder (pull down large values)
  - If a variable is skewed left, move it up the ladder (pull up small values)



Example: Cloud Seeding (Sleuth3 Case Study 3.1.1)

Quote from book: "On each of 52 days that were deemed suitable for cloud seeding, a random mechanism was used to decide whether to seed the target cloud on that day or to leave it unseeded as a control. An airplane flew through the cloud in both cases. . . . [p]recipitation was measured as the total rain volume falling from the cloud base following the airplane seeding run."

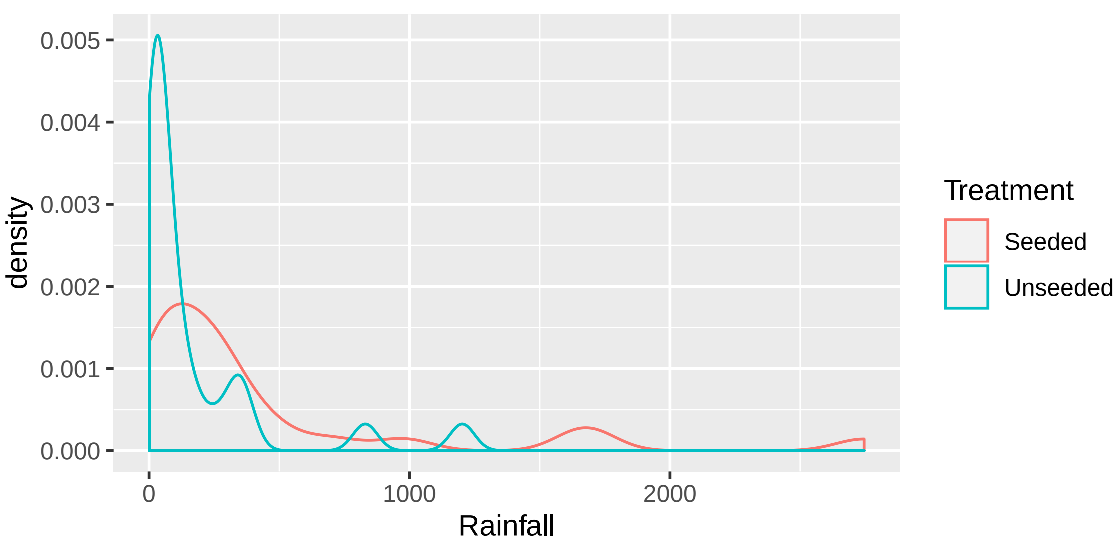
```r
clouds <- read_csv("http://www.evanlray.com/data/sleuth3/case0301_cloud_seeding.csv")
head(clouds)
```

```
## # A tibble: 6 x 2
##   Rainfall Treatment
##      <dbl> <chr>
## 1    1203. Unseeded
## 2     830. Unseeded
## 3     372. Unseeded
## 4     346. Unseeded
## 5     321. Unseeded
## 6     244. Unseeded
```

Starting Point

Here are density plots and box plots, separately for each Treatment.

```r
ggplot(data = clouds, mapping = aes(x = Rainfall, color = Treatment)) +
  geom_density()
```



Standard deviations for each group:

```r
clouds %>%
  group_by(Treatment) %>%
  summarize(
    sd_rainfall = sd(Rainfall)
  )
```

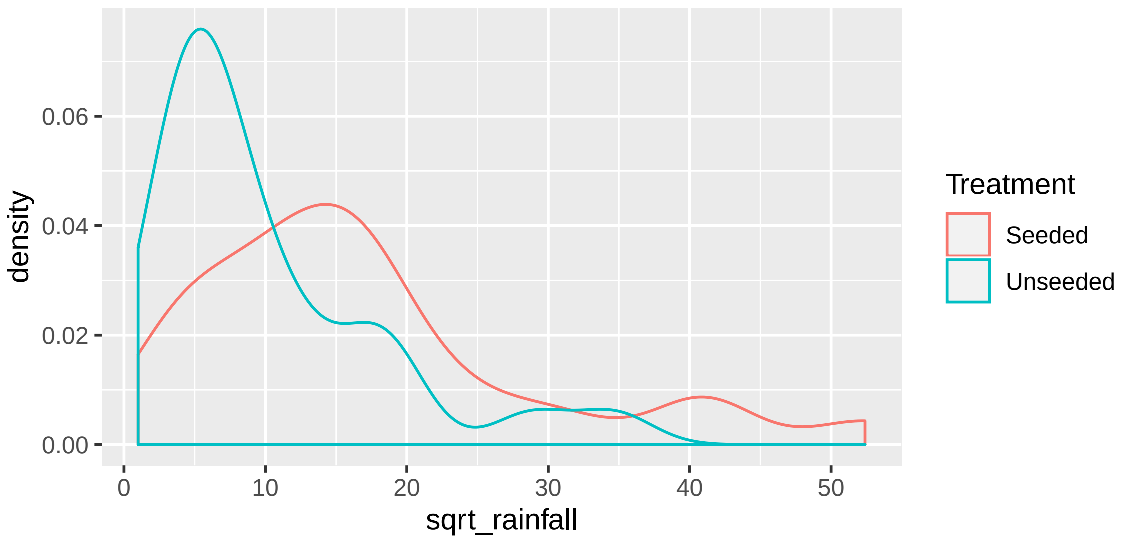
```
## # A tibble: 2 x 2
##   Treatment sd_rainfall
##   <chr>           <dbl>
## 1 Seeded           651.
## 2 Unseeded         278.
```

Skewed right, so move down one step on the ladder.

Down 1 Step: $\sqrt{Rainfall}$

```
clouds <- clouds %>%
  mutate(
    sqrt_rainfall = sqrt(Rainfall)
  )
```

```
ggplot(data = clouds, mapping = aes(x = sqrt_rainfall, color = Treatment)) +
  geom_density()
```




```
clouds %>%
  group_by(Treatment) %>%
  summarize(
    sd_rainfall = sd(sqrt_rainfall)
  )
```

```
## # A tibble: 2 x 2
##   Treatment sd_rainfall
##   <chr>           <dbl>
## 1 Seeded          12.5
## 2 Unseeded         8.24
```

These distributions are closer to symmetric – probably good enough.

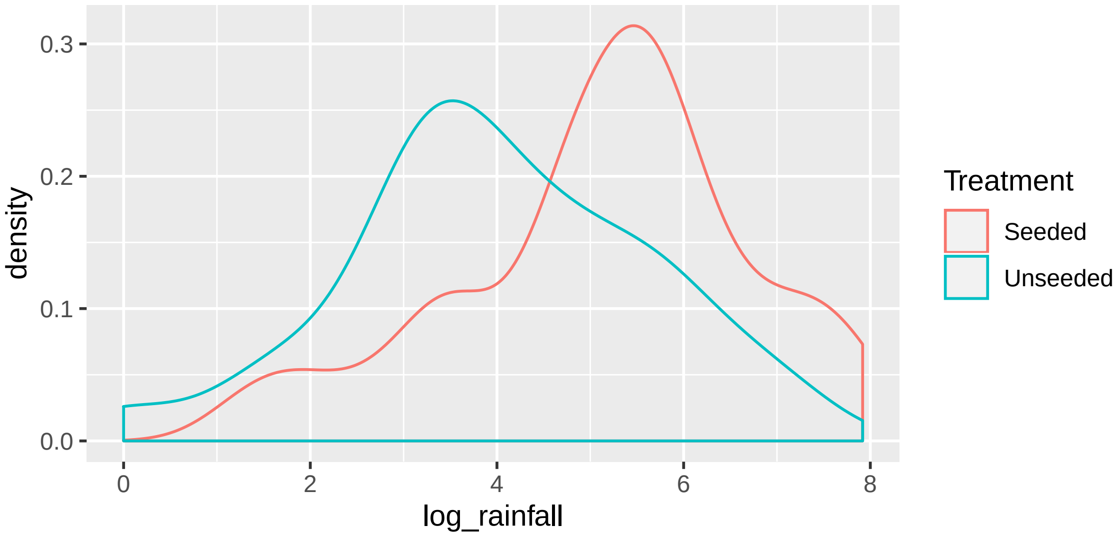
The ratio of these standard deviations is less than 2 – often used as a guide for when we're OK.

However, we can make it even better if we go down another step.

Down 2 Steps: $\log(Rainfall)$

```
clouds <- clouds %>%
  mutate(
    log_rainfall = log(Rainfall)
  )
```

```
ggplot(data = clouds, mapping = aes(x = log_rainfall, color = Treatment)) +
  geom_density()
```




```
clouds %>%
  group_by(Treatment) %>%
  summarize(
    sd_rainfall = sd(log_rainfall)
  )
```

```
## # A tibble: 2 x 2
##   Treatment sd_rainfall
##   <chr>           <dbl>
## 1 Seeded           1.60
## 2 Unseeded         1.64
```

Good enough! We can conduct our analysis on this scale.

Analysis on transformed scale

```
clouds %>%
  group_by(Treatment) %>%
  summarize(
    mean_log_rainfall = mean(log_rainfall)
  )
```

```
## # A tibble: 2 x 2
##   Treatment mean_log_rainfall
##   <chr>                 <dbl>
## 1 Seeded                 5.13
## 2 Unseeded               3.99
```

```
rainfall_fit <- lm(log_rainfall ~ Treatment, data = clouds)
library(gmodels)
fit.contrast(rainfall_fit, "Treatment", c(1, -1), conf.int = 0.95)
```

```
##                      Estimate Std. Error  t value   Pr(>|t|) lower CI
## Treatment c=( 1 -1 ) 1.143781  0.4495342 2.544369 0.01408266 0.240865
##                      upper CI
## Treatment c=( 1 -1 ) 2.046697
## attr(,"class")
## [1] "fit_contrast"
```

We can interpret these numbers either on the new, transformed, data scale or on the original data scale.

1. Interpret the group mean estimates above on the transformed scale (always works!):

2. Interpret the group mean estimates above on the original data scale (works if we got to a place where distributions were approximately symmetric after transformation!):

```
exp(5.13)
```

```
## [1] 169.0171
```

```
exp(3.99)
```

```
## [1] 54.05489
```

```r
rainfall_fit <- lm(log_rainfall ~ Treatment, data = clouds)
summary(rainfall_fit)
```

```
##
## Call:
## lm(formula = log_rainfall ~ Treatment, data = clouds)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.9904 -0.7453  0.1624  1.0187  3.1018
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)          5.1342     0.3179  16.152   <2e-16 ***
## TreatmentUnseeded   -1.1438     0.4495  -2.544   0.0141 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.621 on 50 degrees of freedom
## Multiple R-squared:  0.1146, Adjusted R-squared:  0.09693
## F-statistic: 6.474 on 1 and 50 DF,  p-value: 0.01408
```

```r
confint(rainfall_fit)
```

```
##                       2.5 %    97.5 %
## (Intercept)        4.495729  5.772645
## TreatmentUnseeded -2.046697 -0.240865
```

```r
library(gmodels)
fit.contrast(rainfall_fit, "Treatment", c(1, -1), conf.int = 0.95)
```

```
##                        Estimate Std. Error  t value   Pr(>|t|) lower CI
## Treatment c=( 1 -1 ) 1.143781  0.4495342 2.544369 0.01408266 0.240865
##                        upper CI
## Treatment c=( 1 -1 ) 2.046697
## attr(,"class")
## [1] "fit_contrast"
```

3. Interpret the estimated difference in means above on the transformed scale (always works!):

4. Interpret the estimted difference in means above on the original data scale (works only if the transformation selected was the log transformation and the resulting distribution was approximately symmetric!):

```r
exp(1.143781)
```

```
## [1] 3.138613
```

```r
exp(0.240865)
```

```
## [1] 1.272349
```

```r
exp(2.046697)
```

```
## [1] 7.742286
```